I $\sin\left(-\frac{11\pi}{12}\right)$

$-\sin\left(\frac{11\pi}{12}\right)$

$-\sin\left(\frac{3\pi}{12}+\frac{8\pi}{12}\right)$

$-\sin\left(\frac{\pi}{4}+\frac{2\pi}{3}\right)$

$-\sin\frac{\pi}{4}\cos\frac{2\pi}{3}-\sin\frac{2\pi}{3}\cos\frac{\pi}{4}$

$-\frac{\sqrt{2}}{2}\cdot\left(\frac{-1}{2}\right)-\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}$

5 $\frac{\sqrt{2}-\sqrt{6}}{4}$

II

5 pts ea

$A(x)=\sin x$ D_A $x\in\mathbb{R}$ R_A $|A(x)|\le 1$ 2π odd

$B(x)=\cos x$ D_B $x\in\mathbb{R}$ R_B $|B(x)|\le 1$ 2π even

$C(x)=\tan x$ D_C $x\notin\left\{\frac{(2k+1)\pi}{2}\right\}$ R_C $C(x)\in\mathbb{R}$ π odd

$D(x)=\sec x$ D_D $x\notin\left\{\frac{(2k+1)\pi}{2}\right\}$ R_D $|D(x)|\ge 1$ 2π even

$E(x)=\csc x$ D_E $x\notin\{k\pi\}$ R_E $|E(x)|\ge 1$ 2π odd

$f(x)=\cot x$ D_f $x\notin\{k\pi\}$ R_f $f(x)\in\mathbb{R}$ π odd

III

A) $\frac{\cos 2x}{\cos x}=\frac{1-\tan^2 x}{\sec x}$

$=\frac{1-\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos x}}$

$\cos x\neq 0$

$x\notin\left\{\frac{(2k+1)\pi}{2}\right\}$

$=\frac{\frac{\cos^2 x-\sin^2 x}{\cos^2 x}}{\frac{1}{\cos x}}$

15

$=\frac{\cos 2x}{\cos x}$

B) $\tan\frac{x}{2}=\csc x-\cot x$

$=\frac{1}{\sin\frac{x}{2}+\frac{x}{2}}-\frac{\cos\left(\frac{x}{2}+\frac{x}{2}\right)}{\sin\left(\frac{x}{2}+\frac{x}{2}\right)}$

$\frac{x}{2}+\frac{x}{2}=x$

-6

$=\frac{1-\cos^2\left(\frac{x}{2}\right)+\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$=\frac{\cancel{2}\sin^2\frac{x}{2}}{\cancel{2}\sin\frac{x}{2}\cos\frac{x}{2}}$

15

$=\tan\frac{x}{2}$

$x\notin\{k\pi\}$

C) $\sec^2 x-\cos^2 x=\sin^2 x+\tan^2 x$

$=\sin^2 x+\frac{\sin^2 x}{\cos^2 x}$

$x\notin\left\{\frac{(2k+1)\pi}{2}\right\}$

$=\sin^2 x+\frac{1-\cos^2 x}{\cos^2 x}$

$=\sin^2 x+\frac{1}{\cos^2 x}-1$

15

$=-\cos^2 x+\sec^2 x$

Good effort worth 9 pts

$\sin^2 x+\tan^2 x$

$1-\cos^2 x+\tan^2 x$

$\underbrace{1+\tan^2 x}_{\sec^2 x}-\cos^2 x$

$\sec^2 x-\cos^2 x$

IV $\sin x = \frac{4}{5}$ $\frac{\pi}{2} < x < \pi$ $\cot 2x = ? = \frac{\cos 2x}{\sin 2x} = \frac{\cos^2 x - \sin^2 x}{2\sin x \cos x}$

$\cos x = \frac{-3}{5}$ $\cot 2x = \frac{\frac{9}{25} - \frac{16}{25}}{2 \cdot \frac{4}{5}\left(\frac{-3}{5}\right)} = \frac{\frac{-7}{25}}{\frac{-24}{25}} = \frac{7}{24}$. 10

V $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^3 x}$

Do not use cos, sin add -4

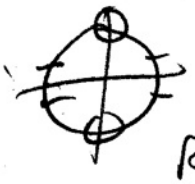
$\tan 2x + x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

$= \frac{\frac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \frac{2\tan x}{1-\tan^2 x} \cdot \tan x}$

$= \frac{\frac{2\tan x + \tan x - \tan^3 x}{\cancel{1-\tan^2 x}}}{\frac{1 - \tan^2 x - 2\tan^2 x}{\cancel{1-\tan^2 x}}}$

10

$= \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$



Restrictions not required.

$3\tan^2 x = 1$

$\tan x = \pm\frac{1}{2}$, $\pm\frac{\sqrt{3}}{2}$

$3x \neq \frac{(2k+1)\pi}{2}$

$x \neq \frac{(2k+1)\pi}{6}$

$\sin(4x+2x)$

$4x-2x$

Extra

$\sin 6x + \sin 2x = 2\sin 4x \cos 2x$ $x \in \mathbb{R}$

$\sin(4x+2x) +$

$\sin 4x \cos 2x + \sin 2x \cos 4x + \sin 2x =$

$+ \sin 2x(\cos 4x + 1) =$

$+ \sin 2x(\cos^2 2x - \sin^2 2x + 1) =$ $\cos^2 x$

$+ 2\sin 2x \cos 2x \cos 2x =$

$+ \sin 4x \cos 2x =$

$\sin 4x \cos 2x = 2\sin 4x \cos 2x$

1993

	2	5	T
100	1	2	3
90	3	7	10
80	6	5	11
70	6	4	10
60	5	3	8
54	3	3	6

102

47

$\frac{1825}{24}$ $\frac{1833}{24}$

76.0 76.4 76.2

Excellent exam — still!
I need to adjust seating —
too much inattention
too much chit chat.
3 of top 4 are repeat
students!

2000

	4	5	T
100	1	4	
90	4	7	
80	6	8	
70	6	4	
60	4	4	
54	4	3	

104

$\frac{1835}{24}$ $\frac{2408}{29}$ $\frac{4243}{53}$ 61

76 83.0 80.1

I was out 2 weeks
However I still cannot
explain why they refused
to learn $\frac{x}{2}+\frac{x}{2}=x$

and many lost points on
Part I. They didn't
put the reduction ideas
together.

[redacted]

1996

100%	3
90%	10
80%	7
70%	8
60%	5
59	1

105

59

$\frac{2821}{34}$

82.97

83.0

Lowest students capied work from boards in spite of constant harassment from me. maybe Now they'll behave....

DIDN'T use $\frac{X}{2}+\frac{Y}{2}$ on [illegible] B cost 10 pts.

errors in proof which "somehow" resulted in identities cost 8 pts.

Nobody did extra as was shown in class previously.

Honors Trigonometry

Name Key Ch 2

Per _____ Date 1996

43/96, 2000

Copy the original problem.
Convince *me* that you understand the concept.
No decimal answers.

I Determine the exact value of the following. Reduce fully.

Do not leave radicals in the denominator.: $\csc\left(\frac{-41\pi}{12}\right)$ (10 pts)

II For each of the following functions, state the domain, range, period and state whether the function is odd, even, or neither. (25 pts tot)

A) $A(x) = \sin x$ B) $B(x) = \cos x$ C) $C(x) = \tan x$

D) $D(x) = \sec x$ E) $E(x) = \csc x$ F) $F(x) = \cot x$

III State the sine, cosine, and tangent "add/subtract" formulas we use in class.(10 pts tot)

IV Leaving the *LEFT SIDE UNTOUCHED*, **PROVE** the given identity. (~~10~~ 15 pts ea)

A) $\frac{\cos 2x}{\cos x} = \frac{1 - \tan^2 x}{\sec x}$ B) $\tan\left(\frac{x}{2}\right) = \csc x - \cot x$

C) $\sec^2 x - \cos^2 x = \sin^2 x + \tan^2 x$

V Given $\sin x = \frac{4}{5}$ and $\frac{\pi}{2} < x < \pi$. Find $\cot 2x$. (10 pts)

EXTRA CREDIT ..... **5 pts** .....

Prove: $\sin 6x + \sin 2x = 2 \sin 4x \cos 2x$

I

$\sec\left(-\frac{13\pi}{12}\right)$

Sec even

$\sec\left(\frac{13\pi}{12}\right)$

$\frac{1}{\cos\left(\frac{9\pi}{12}+\frac{4\pi}{12}\right)}$

$\frac{3\pi}{4}+\frac{\pi}{3}$

No cos Add

−7

$\frac{1}{\cos\frac{3\pi}{4}\cos\frac{\pi}{3}-\sin\frac{3\pi}{4}\sin\frac{\pi}{3}}$

$\frac{1}{-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}}$

$\frac{1}{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

Stopping −2

$\frac{-4}{\sqrt{2}+\sqrt{6}}$ $\quad \frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}}$

$\frac{-4(\sqrt{2}-\sqrt{6})}{2-6}$

$\sqrt{2}-\sqrt{6}$ using sine $\sqrt{6}+\sqrt{2}$

II

$A(x)=\sin x$

D_A $x\in\mathbb{R}$

R_A $-1\le\sin x\le1$

Per 2π

odd

$B(x)=\cos x$

D_B $x\in\mathbb{R}$

R_B $-1\le\cos x\le1$

Per 2π

Even

$C(x)=\tan x$

D_C $x\neq\frac{(2k+1)\pi}{2}$

R_C $\tan x\in\mathbb{R}$

Per π

odd

$D(x)=\sec x$

D_D $x\neq\left\{\frac{(2k+1)\pi}{2}\right\}$

R_D $\sec x\le-1$ or $\sec x\ge1$

Per 2π even

$E(x)=\csc x$

D_E $x\neq\{k\pi\}$

R_E $\csc x\le-1$ or $\csc x\ge1$

Per 2π odd

$f(x)=\cot x$

D_f $x\neq\{k\pi\}$

R_f $\cot x\in\mathbb{R}$

Per π

odd

III

$\sin(A\pm B)=\sin A\cos B\pm\sin B\cos A$

$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$

$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$

IV

A) $\frac{\csc x}{2\cos x}=\csc 2x$

$=\frac{1}{2\sin x\cos x}\quad\frac{\frac{1}{\sin x}}{\frac{1}{\sin x}}$

$=\frac{\csc x}{2\cos x}$

B) $\sec x\csc x=2\csc 2x$

$=\frac{\cancel{2}}{\cancel{2}\sin x\cos x}$

$=\frac{1}{\sin x}\cdot\frac{1}{\cos x}$

$=\csc x\cdot\sec x$

C) $\sin^2\frac{x}{2}=\frac{1}{2}(1-\cos x)$

$=\frac{1}{2}\left(1-\left[\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right]\right)$

$=\frac{1}{2}\left(1-\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\right)$

$=\frac{1}{2}\left(2\sin^2\frac{x}{2}\right)$

$=\sin^2\frac{x}{2}$

No $\frac{x}{2}+\frac{x}{2}=-6$

magic −8

IV D) $\dfrac{3\tan x - \tan^3 x}{1 - 3\tan x} = \tan 3x$ $(2x+x)$

$$= \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

$$= \frac{\dfrac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \dfrac{2\tan^2 x}{1-\tan^2 x}}$$

$$= \frac{2\tan x + \tan x - \tan^3 x}{1 - \tan^2 x - 2\tan^2 x}$$

$$= \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

V

$\tan x = 3$

$\pi < x < \frac{3\pi}{2}$

$\sec 2x =$

$\dfrac{1}{\cos 2x}$

$\dfrac{1}{\cos^2 x - \sin^2 x}$ $\dfrac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x}}$

$\dfrac{\sec^2 x}{1 - \tan^2 x}$

$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2 x$

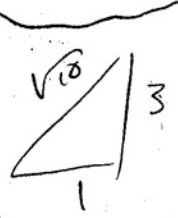
$9 + 1 = \sec^2 x$

$\dfrac{10}{1-9}$

$-\dfrac{10}{8} = -\dfrac{5}{4}$

Effort but no simplify -4

Not shown at this time



$\cos 2x$

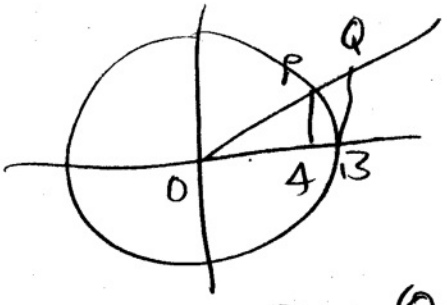
$\cos^2 x - \sin^2 x$

$\frac{1}{10} - \frac{9}{10}$

$-\frac{8}{10}$

$-\frac{4}{5}$ so $\sec 2x = -\frac{5}{4}$

EXTRA



	O	$(0,0)$
	A	$(0, \cos x)$
	B	$(1,0)$
2 PTS	P	$(\cos x, \sin x)$
5 PTS	Q	$(1, \tan x)$

10/7/98

	4	5	+
100	1	1	
90	6	5	
80	7	17	
70	9	1	
60	1	3	
59	1	0	

102

2060	2272	4332
25	27	52
82.4	84.1	83.3

51

10/3/2001

	3	5	
100	2	1	105
90	6	2	
80	8	8	
70	5	8	
60	3	8	
59	1	2	

2051	2202
25	29
82.0	75.9

54

78.8

still a great exam!!

95

	5	6	T
100	1	1	
90	8	6	
80	10	9	
70	5	7	
60	2	4	
59	3	2	

100

47

$\frac{2373}{29}$ $\frac{2227}{29}$ $\frac{4600}{58}$

81.83 76.79 79.3

Excellent exam!
Students are copying from the boards and now maybe they will see what being unprepared is all about —
No changes.

Honors Trigonometry

Name Trig Ch 2

Per _____ Date 10/10/95/98/01

Copy the original problem.
Convince *me* that **you** understand the concept.
No decimal answers.

I Determine the exact value of the following. Reduce fully.

Do not leave radicals in the denominator.: $\sec\left(\frac{-13\pi}{12}\right)$ (10 pts)

II For each of the following functions, state the domain, range, period and state whether the function is odd, even, or neither. (5 pts ea)

A) $A(x) = \sin x$ B) $B(x) = \cos x$ C) $C(x) = \tan x$

D) $D(x) = \sec x$ E) $E(x) = \csc x$ F) $F(x) = \cot x$

III State the sine, cosine, and tangent "add/subtract" formulas we use in class.(10 pts tot)

IV Leaving the *LEFT SIDE UNTOUCHED*, **PROVE** the given identity.

(10 pts ea)

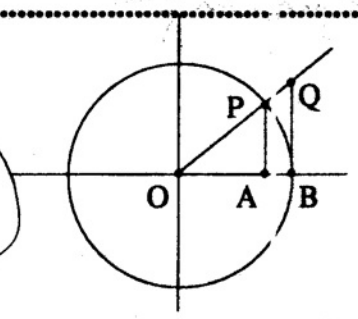
A) $\frac{\csc x}{2\cos x} = \csc 2x$ B) $\sec x \csc x = 2\csc 2x$

C) $\sin^2 \frac{x}{2} = \frac{1}{2}(1 - \cos x)$ D) $\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} = \tan 3x$

V Given $\tan x = 3$ and $\pi < x < \frac{3\pi}{2}$. Find $\sec 2x$. (10 pts)

EXTRA CREDIT ..... **5 pts** .....

Given QB is tangent to the unit circle at B.
The arc length PB is x. Determine the coordinates of **all 5 points** indicated with letters.



I $\sec\left(\frac{-37\pi}{12}\right)$

$\sec\left(\frac{37\pi}{12}\right)$

$\sec\left(\frac{13\pi}{12}+\frac{24\pi}{12}\right)$

$\frac{1}{\cos\frac{13\pi}{12}}$

$\cos\left(\frac{9\pi}{12}+\frac{4\pi}{12}\right)$

$\left(\frac{3\pi}{4}+\frac{\pi}{3}\right)$

w/o circle 2pt per error.

$\cos\frac{3\pi}{4}\cos\frac{\pi}{3}-\sin\frac{\pi}{3}\sin\frac{3\pi}{4}$

$-\frac{\sqrt{2}}{2}\,\frac{1}{2}-\frac{\sqrt{3}}{2}\,\frac{\sqrt{2}}{2}$

$\frac{-\sqrt{2}-\sqrt{6}}{4}$ = cos

Reciprocal.

$\frac{-4}{\sqrt{2}+\sqrt{6}}$

II 6 trig fctns D, R, Period/Zeros 2 pts ± off -3

III 3 Add formulas 5 pts ea No partial credit.

IV

A) $\sec x\csc x = \tan x + \cot x$

$= \frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}$

$= \frac{\sin^2 x+\cos^2 x}{\sin x\cos x}$

$= \frac{1}{\sin x\cos x}$

$\sec x\csc x = \csc x\sec x$

miracle proofs at least 7 pts

B) $2\sin 2x\cos x = \sin 3x+\sin x$

$= \sin 2x\cos x+\sin x\cos x+\sin x$

$+\sin x(\cos^2 x-\sin^2 x+1)$

$+\sin x(\cos^2 x+\cos^2 x)$

$+\underbrace{2\sin x\cos x}_{\sin 2x}\cos x$

$= 2\sin 2x\cos x$

C) $3\cos^2 x = \cos 2x+\cos^2 x+1$

$= \cos^2 x-\sin^2 x+\cos^2 x+1$

$= 3\cos^2 x$

D) $\frac{3\tan x-\tan^2 x}{1-3\tan^2 x} = \tan 3x$

$= \frac{\tan 2x+\tan x}{1-\tan 2x\tan x}$

$= \frac{\frac{2\tan x}{1-\tan^2 x}+\tan x}{1-\frac{2\tan x}{1-\tan^2 x}\cdot\tan x}$

$= \frac{\frac{2\tan x+\tan x-\tan^3 x}{1-\tan^2 x}}{1-\frac{2\tan^2 x}{1-\tan^2 x}}$

$\frac{\frac{3\tan x-\tan^3 x}{\cancel{1-\tan^2 x}}}{\frac{1-\tan^2 x-2\tan^2 x}{\cancel{1-\tan^2 x}}}$

$= \frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$

V $\sin x = \frac{-4}{5}$

$\cot 2x = \frac{1}{\tan 2x} = \frac{1}{\frac{2\tan x}{1-\tan^2 x}}$

$\cos^2 x = 1 - \frac{16}{25}$

$\cos^2 x = \frac{9}{25}$

$\cos x = \frac{3}{5}$ 1st Quad

$\tan x = \frac{-\frac{4}{5}}{\frac{3}{5}} = -\frac{4}{3}$

$= \frac{1-\tan^2 x}{2\tan x}$

$= \frac{1 - \frac{16}{9}}{2\left(\frac{-4}{3}\right)}$

$\frac{2\cos x}{2\sin x} = -4$

$\frac{\frac{-7}{9}}{\frac{-8}{3}}$ $\left(\frac{-7}{9}\right)\left(\frac{-3}{8}\right)$

$\frac{7}{24} = \cot 2x$

Extra.

$\sin 6x + \sin 2x = 2\sin 4x \cos 2x$

$\sin(4x+2x) + \sin(4x-2x)$

$\sin 4x \cos 2x + \underbrace{\sin 2x \cos 4x + \sin 4x \cos 2x - \sin 2x \cos 4x}_{\text{zero}}$

$\Downarrow$

$2\sin 4x \cos 2x = 2\sin 4x \cos 2x$

	2	3
100	4	1
90	8	2
80	5	7
70	2	7
60	6	5
59	2	6

105

35

$\frac{2239}{27}$ $\frac{2042}{28}$

829 729 77.8

Excellent —
EASY exam!
Some don't prep....
oh well

Honors Trigonometry

Name TRIG Ch2 EXAM

Per _____ Date _____

Copy the original problem.
Convince *me* that **you** understand the concept.
No calculators, of course.

II 10x
I 20x

2002

I Determine the exact value of the following. Reduce fully.

You may leave radicals in the denominator.: $\sec\left(\frac{-37\pi}{12}\right)$ (10 pts)

II For each of the six standard trig functions, state the domain, range, period and state whether the function is odd, even, or neither. (25 pts tot)

III State the sine, cosine, and tangent "add/subtract" formulas we use in class.(15 pts tot)

IV Leaving the *LEFT SIDE UNTOUCHED,* **PROVE** the given identity.

(10 pts ea)

A) $\sec x \csc x = \tan x + \cot x$

B) $2\sin 2x \cos x = \sin 3x + \sin x$

C) $3\cos^2 x = \cos 2x + \cos^2 x + 1$

D) $\dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} = \tan 3x$

V Given $\sin x = \frac{-4}{5}$ and $\frac{3\pi}{2} < x < 2\pi$. Find $\cot 2x$. (10 pts)

EXTRA CREDIT ..... **5 pts** .....

Prove: $\sin 6x + \sin 2x = 2\sin 4x \cos 2x$

***FINALLY*, WE GET TO DO SOME *REAL* TRIGONOMETRY !!!**		EXAM CHAPTER 1 9/18	**#1** "Circle Secrets" Section I 9/19	**#2** "Circle Secrets" Section II 9/20
# 3 Circle Secrets III Pg 31 17, 18, 23, 24 note: sine is >0 & cosine < 0 in only 2nd quad **# 4** Pg 26 1 - 7, for 1 - 6 use cos add/sub formula then on pblm 7 ignore "properties" 9/ 23	**#5** "Circle Secrets" Section IV, V 9/24	**#6** "Circle Secrets" Section VI Pg 55 18 – 23 Radical in Denom will be ok. No graphing. 9/25	**# 7** Pg 71 1 – 13 9/26	**# 8** Pg 71 14 – 18 **#9** Pg 40 7 – 12 **We *always* prove!** 9/27
# 10 Pg 40 13 – 16 **# 11** Pg 67 1 – 11 9/30	**#12** "Circle Secrets" Section VII Quiz? 10/1	**#13** Pg 71 - 72 22,23, 30, 31, 32, 34 10/2	**#14** R/C Pg 44 **#15** Pg 67 14, 15, 17 10/3	**# 16** "Circle Secrets" Section VIII 10/4
# 17 Pg 45 2 - 6, 8 Pg 73 1, 2, 4, 5 10/ 7	Put Last Year's Exam on the Boards. 10/8	EXAM **CHAPTER 2** 2002 10/ 9	***We Love Trig !***	

I $\cos\left(-\frac{7\pi}{12}\right) = \cos\left(\frac{7\pi}{12}\right)$

$\cos\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$\frac{\sqrt{2}}{2}\,\frac{1}{2} - \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2}$

$\frac{\sqrt{2} - \sqrt{6}}{4}$

II

A) $A(x) = \sin x$
D_A $x \in \mathbb{R}$
R_A $|\sin x| \le 1$
odd

B) $B(x) = \cos x$
D_B $x \in \mathbb{R}$
R_B $|\cos x| \le 1$
Even

C) $C(x) = \tan x$
D_C $x \notin \left\{\frac{(2k+1)\pi}{2}\right\}$
R_C $C(x) \in \mathbb{R}$
odd

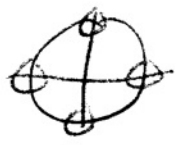
D) $D(x) = \sec x$
D_D $x \notin \left\{\frac{(2k+1)\pi}{2}\right\}$
R_D $|\sec x| \ge 1$
Even

E) $E(x) = \csc x$
D_E $x \notin \{k\pi\}$
R_E $|\csc x| \ge 1$
odd

F) $f(x) = \cot x$
D_f $x \notin \{k\pi\}$
R_f $f(x) \in \mathbb{R}$
odd

III

A) $\sec x \csc x = \tan x + \cot x$



$x \notin \left\{\frac{k\pi}{2}\right\}$

$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

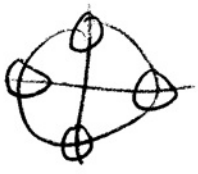
$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$

$= \frac{1}{\cos x \sin x}$

$= \sec x \csc x$

III

B) $\tan x = \frac{1 - \cos 2x}{\sin 2x}$



$x \notin \left\{\frac{k\pi}{2}\right\}$

$= \frac{1 - \cos^2 x + \sin^2 x}{2\sin x \cos x}$

$= \frac{2\sin^2 x}{2\sin x \cos x}$

$= \frac{\sin x}{\cos x}$

$= \tan x$

C) $1 - \sin x = \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2$

$= \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2} + \sin^2\frac{x}{2}$

$= 1 - 2\sin\frac{x}{2}\cos\frac{x}{2}$

$= 1 - \sin x$

$x \in \mathbb{R}$

IV

$\sin x = \frac{4}{5}$ $\quad \frac{\pi}{2} < x < \pi$ $\quad \csc 2x = ?$

$s^2 + c^2 = 1$

$\frac{16}{25} + c^2 = 1$

$c = \pm \frac{3}{5} \rightarrow \cos x = -\frac{3}{5}$

$\csc 2x = \frac{1}{2 \sin x \cos x}$

$\therefore \frac{1}{2 \cdot \frac{4}{5} \cdot \left(\frac{-3}{5}\right)} = \frac{-1}{\frac{24}{25}}$

$= \frac{-25}{24}.$

V

A) f has period a if $f(x+a) = f(x)$

B) $f(x) = \tan x$

$f(x+\pi) = \tan(x+\pi)$

$= \frac{\tan x + \tan \pi}{1 - \tan x \tan \pi}$

$= \frac{\tan x + 0}{1 - \tan x \cdot 0}$

$= \tan x$

$= f(x)$

$\therefore$ f has period π.

Extra:

$2\cos^2 x + 3\cos x = -1$

$2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$2\cos x + 1 = 0 \quad \cos x = -1$

$\cos x = -\frac{1}{2}$

$x \in \left\{ \frac{2\pi}{3}, \pi, \frac{4\pi}{3} \right\}$

1986

	2	5	T
100	1	4	5
90	8	8	16
80	7	9	16
70	7	3	10
60	4	5	9
54	2	2	4
	80.8	82.6	81.8

103

50

excellent exam —
changes indicated
would be final.

the "I grade it" quiz
#2 R, 1 of factors
is vital.

Quizzes helped —
after 1st one I said you
know the answers quite em.

	2	5
100	0	2
90	10	16
80	4	2
70	1	7
60	0	1
59	0	0

101

65

90.1 91.91 91.2

$\frac{1352}{15}$ $\frac{2114}{23}$ $\frac{3466}{38}$

Honors Trigonometry Name TRIG QIT2 91

Copy the original problem.
Convince *me* that **you** understand the concept. Per _____ Date _____

Exam # 2 Quarter # 1

I Evaluate: $\sin\left(\frac{-7\pi}{12}\right)$ (5 pts)

II For each of the following functions, state the domain, range, period and state whether the function is odd, even, or neither. (5 pts ea)

A) A(x) = Sin x B) B(x) = Cos x C) C(x) = Tan x

D) D(x) = Sec x E) E(x) = Csc x F) F(x) = Cot x

III Leaving the *LEFT SIDE UNTOUCHED*, **PROVE** the given identity.
List restrictions using the "k" notation. (15 pts ea)

A) $\sec x \ \csc x = \tan x + \cot x$

B) $\tan x = \dfrac{1-\cos 2x}{\sin 2x}$

C) $3\cos^2 x = \cos 2x + \cos^2 x + 1$

IV Given: Sin x = $\frac{4}{5}$ and $\frac{\pi}{2} < x < \pi$. Find the value of Sin 3x. (10 pts)

V Prove: $1 + \dfrac{1}{\cos x} = \dfrac{\tan^2 x}{\sec x - 1}$. Disregard restrictions. (10 pts)

EXTRA CREDIT ═══ 5 pts ═══

Determine all $x \ \varepsilon [0, 2\pi)$ (i.e. $0 \le x < 2\pi$) where the following is true.
Write your answers as elements of a set and mark the locations on a unit circle.

$$\cos x(2\cos x + 3) = -1$$

I

$\cos\left(\frac{-\pi}{12}\right)$

$\cos\left(\frac{\pi}{12}\right)$

$\cos\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right)$

$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$\frac{1}{2}\,\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}+\sqrt{6}}{4}$

II

$A(x)=\sin x$ $\quad B(x)=\cos x$ $\quad C(x)=\tan x$

D_A $x\in\mathbb{R}$ $\quad D_B$ $x\in\mathbb{R}$ $\quad D_C$ $x\notin\left\{\frac{(2k+1)\pi}{2}\right\}$

R_A $|A(x)|\le 1$ $\quad R_B$ $|B(x)|\le 1$ $\quad R_C$ $C(x)\in\mathbb{R}$

Per 2π $\quad$ Per 2π $\quad$ Per π

odd $\quad$ even $\quad$ odd

$D(x)=\sec x$ $\quad E(x)=\csc x$ $\quad F(x)=\cot x$

D_D $x\notin\left\{\frac{(2k+1)\pi}{2}\right\}$ $\quad D_E$ $x\notin\{k\pi\}$ $\quad D_F$ $x\notin\{k\pi\}$

R_D $|D(x)|\ge 1$ $\quad R_E$ $|E(x)|\ge 1$ $\quad R_F$ $F(x)\in\mathbb{R}$

Per 2π $\quad$ Per 2π $\quad$ Per π

even $\quad$ odd $\quad$ odd

-1 cosmetic error $\quad -2$ wrong answer.

III

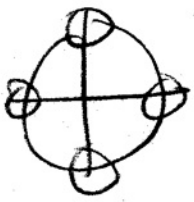
A) $\sin(A\pm B)=\sin A\cos B\pm\sin B\cos A$

C) $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$

D) $\tan(A\pm B)=\dfrac{\tan A\pm\tan B}{1\mp\tan A\tan B}$ 4pts

IV

A) $\tan x=\dfrac{1-\cos 2x}{\sin 2x}$



$x\notin\left\{\frac{k\pi}{2}\right\}$

$=\dfrac{1-(\cos^2x-\sin^2x)}{2\sin x\cos x}$

$=\dfrac{1-\cos^2x+\sin^2x}{2\sin x\cos x}$

$=\dfrac{\cancel{2}\sin^2x}{\cancel{2}\sin x\cos x}$

$\tan x=\tan x$

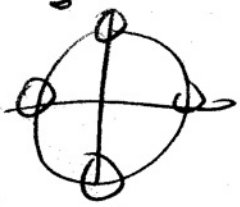
B) $\sec x\csc x=2\csc 2x$

$=\dfrac{2}{\sin 2x}$

$=\dfrac{\cancel{2}}{\cancel{2}\sin x\cos x}$

$=\dfrac{1}{\sin x\cos x}$

$\sec x\csc x=\sec x\csc x$



$x\notin\left\{\frac{k\pi}{2}\right\}$

IV

c)

$$\sin^2\frac{x}{2} = \frac{1}{2}(1-\cos x)$$
$$= \frac{1}{2}\left(1-\cos\left(\frac{x}{2}+\frac{x}{2}\right)\right)$$
$$= \frac{1}{2}\left(1-\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\right)$$
$$= \frac{1}{2}\left(2\sin^2\frac{x}{2}\right)$$
$$= \sin^2\frac{x}{2}$$

$x \in \mathbb{R}$.

10 pts diff. method used if ~~omitted~~ and right answer

−12

If diff method used ~~No conclusion~~ and NOT $\sin^2\frac{x}{2}$ end

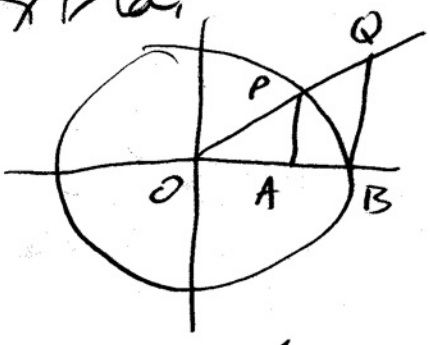
−8 or −10 depending on effort.

V $f(x+a)=f(x)$ ← Explain what you are doing.

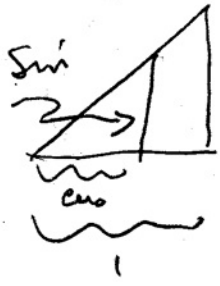
$$\tan(x+\pi) = \tan x$$
$$\frac{\tan x + \overset{0}{\tan\pi}}{1-\tan x\,\underset{0}{\tan\pi}}$$
$$\tan x = \tan x.$$

tan has period π ← conclusion 1pt.

Extra



O $(0,0)$
A $(\cos x, 0)$
B $(0,1)$
P $(\cos x, \sin x)$
Q $(1, \tan x)$



?

$$\frac{?}{1} = \frac{\sin x}{\cos x}$$

1992

	2	5	T
100	4	6	10
90	5	6	11
80	8	6	14
70	3	2	5
60	2	2	4
59	1	2	3

104

47

84.96 86.88 85.94

B. Thom 3 don't even know room number.

Excellent, easy exam. Sets the mood for intro to graphing.

Practically everyone thought this was easier than previous exams.

Honors Trigonometry

Copy the original problem.
Convince *me* that **you** understand the concept.

Name Trig C2 92

Per _____ Date _____

Exam # 2 Quarter # 1

I Evaluate: $\cos\left(-\frac{\pi}{12}\right)$ (5 pts)

II For each of the following functions, state the domain, range, period and state whether the function is odd, even, or neither. (5 pts ea)

A) A(x) = Sin x B) B(x) = Cos x C) C(x) = Tan x

D) D(x) = Sec x E) E(x) = Csc x F) F(x) = Cot x

III Addition Formulas (total 10 pts)

A) State the sine add/subtract formula.

B) State the cosine add/subtract formula.

C) State the tangent add/subtract formula.

IV Leaving the *LEFT SIDE UNTOUCHED*, **PROVE** the given identity.
List restrictions using the "k" notation. (15 pts ea)

A) $\tan x = \dfrac{1-\cos 2x}{\sin 2x}$

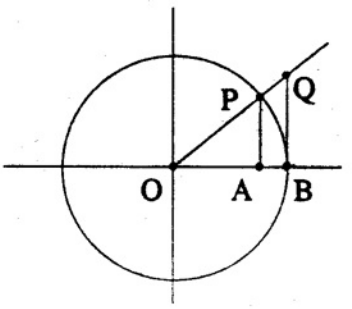
B) $\sec x \csc x = 2\csc 2x$

C) $\sin^2 \frac{x}{2} = \frac{1}{2}(1-\cos x)$

V Prove tan x has period π. (10 pts)

EXTRA CREDIT ═══ 5 pts ═══

Given QB is tangent to the unit circle at B. The arc length PB is x. Determine the coordinates of all 5 points indicated with letters.



I $\csc\left(\frac{-7\pi}{12}\right)$

$$\frac{1}{\sin\left(\frac{3\pi}{12}-\frac{10\pi}{12}\right)}$$

$$\frac{\pi}{4}-\frac{5\pi}{6}$$

$$\frac{1}{\sin\frac{\pi}{4}\cos\frac{5\pi}{6}-\sin\frac{5\pi}{6}\cos\frac{\pi}{4}}$$

$$\frac{1}{\frac{\sqrt{2}}{2}\left(-\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}$$

$$\frac{-\sqrt{6}-\sqrt{2}}{4}$$

$$\frac{4}{-\sqrt{6}-\sqrt{2}}\cdot\frac{-\sqrt{6}+\sqrt{2}}{-\sqrt{6}+\sqrt{2}}$$

$$\frac{4(-\sqrt{6}+\sqrt{2})}{6-2}$$

$$\sqrt{2}-\sqrt{6}$$

II

$A(x)=\sin x$
D_A $x\in\mathbb{R}$
R_A $|\sin x|\le 1$
Per 2π odd

$B(x)=\cos x$
D_B $x\in\mathbb{R}$
R_B $|\cos x|\le 1$
Per 2π even

$C(x)=\tan x$
D_C $x\neq\left\{\frac{(2k+1)\pi}{2}\right\}$
R_C $\tan x\in\mathbb{R}$
Per π odd

$D(x)=\sec x$
D_D $x\neq\left\{\frac{(2k+1)\pi}{2}\right\}$
R_D $|\sec x|\ge 1$
Per 2π even

$E(x)=\csc x$
D_E $x\neq\{k\pi\}$
R_E $|\csc x|\ge 1$
Per 2π odd

$F(x)=\cot x$
D_F $x\neq\{k\pi\}$
R_F $\cot x\in\mathbb{R}$
Per π odd

III

$\sin(A\pm B)=$
$\sin A\cos B\pm\sin B\cos A$

$\cos(A\pm B)=$
$\cos A\cos B\mp\sin A\sin B$

$\tan(A\pm B)=$
$$\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

IV a) $3\cos^2 x=\cos 2x+\cos^2 x+1$

$$\cos^2 x-\sin^2 x+\cos^2 x+1$$

$$\lfloor\cos^2 x\rfloor$$

$$=3\cos^2 x$$

b) $\tan x=\frac{1-\cos 2x}{\sin 2x}$

$$=\frac{1-(\cos^2 x-\sin^2 x)}{2\sin x\cos x}$$

$$=\frac{1-\cos^2 x+\sin^2 x}{2\sin x\cos x}$$

$$=\frac{\sin^2 x+\sin^2 x}{2\sin x\cos x}$$

$$=\frac{\cancel{2}\sin^2 x}{\cancel{2}\sin x\cos x}$$

$$=\tan x$$

IV c) $1+\frac{1}{\cos x}=\frac{\tan^2 x}{\sec x-1}$

$$=\frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos x}-1}$$

$$=\frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1-\cos x}{\cos x}}$$

$$\frac{\sin^2 x}{\cos^2 x}\cdot\frac{\cos x}{1-\cos x}$$

$$\frac{(1-\cos^2 x)}{\cos^2 x}\cdot\frac{\cos x}{1-\cos x}$$

$$\frac{1+\cos x}{\cos x}$$

$$=\frac{1}{\cos x}+1$$

IV work but incomplete or wrong −5 major alg error −7

IV D

$$\frac{3\tan x - \tan^3 x}{1-3\tan^2 x} = \tan 3x$$

$$= \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

$$= \frac{\frac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \frac{2\tan x}{1-\tan^2 x}\tan x}$$

$$= \frac{\frac{2\tan x + \tan x - \tan^3 x}{\cancel{1-\tan^2 x}}}{\frac{1-\tan^2 x - 2\tan^2 x}{\cancel{1-\tan^2 x}}}$$

$$= \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$$

V

$\sin x = \frac{4}{5}$

x in 2nd Quad

$s^2 + c^2 = 1$

$\cos^2 = \frac{9}{25}$

$\cos x = -\frac{3}{5}$

$\sec 3x =$

$$\frac{1}{\cos 3x} =$$

$$\frac{1}{\cos(2x+x)} =$$

$$\frac{1}{\cos 2x \cos x - \sin 2x \sin x}$$

$$\frac{1}{(\cos^2 x - \sin^2 x)\cos x - 2\sin^2 x \cos x}$$

$$\frac{1}{\left(\frac{9}{25} - \frac{16}{25}\right)\left(-\frac{3}{5}\right) - 2\cdot\frac{16}{25}\cdot\left(-\frac{3}{5}\right)}$$

$$\left(\frac{-7}{25}\right)\left(-\frac{3}{5}\right)$$

$$\frac{+21}{125} + \frac{96}{125}$$

$$\frac{117}{125}$$

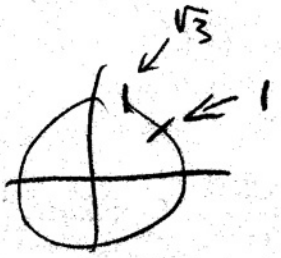
$$\sec 3x = \frac{125}{117}$$

Extra

$\tan 15^\circ$

$$1^\circ = \frac{\pi}{180}$$

$$15^\circ = \frac{36\pi}{12} = \frac{\pi}{12}$$



$$\tan\frac{\pi}{12} = \tan\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$$

$$\frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$$

$$\frac{\sqrt{3}-1}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}}$$

$$\frac{-3+2\sqrt{3}-1}{1-3}$$

$$\frac{4+2\sqrt{3}}{-2}$$

$$-2-\sqrt{3}$$

$$(\sqrt{3}-1)(\sqrt{3}-1)$$

$$(3-2\sqrt{3}+1)$$

change right side of IV D to just Ton 3 X

change left side as indicated

I'd say my error on IV D raised grades about 4 pts over all in the average.

1994

	5	6	T
100	0	1	1
90	9	2	11
80	12	11	23
70	2	5	7
60	0	1	1
59	0	0	0

102

$\frac{1997}{23}$ $\frac{1643}{20}$ $\frac{8417}{60}$

86.8 82.2

I generally took 5 pts if they got lost in IV or 3 if they over looked something.

Note! No restrictions discussed. I'll cover that properly in chapt 4

1997

100↑	3		
90↑	9		
80↑	10		
70↑	4		
60↑	2		
59↓	3		

104

54

$$\frac{2621}{31} = 84.5$$

Excellent exam ---
much better students
than in the past 5 years!
refreshing.

Took 1 pt for giving $\cos x = -\frac{3}{5}$ in $\overline{\text{IV}}$
even though it didn't make any difference
in final answer.

1999

	4	5	T
100↑	0	2	2
90↑	7	8	15
80↑	8	9	17
70↑	6	3	9
60↑	5	2	7
59	0	1	1

104

$\frac{2213}{26}$ $\frac{2136}{25}$ $\frac{4349}{51}$ 50

85.1 85.4 85.3

Great exam!

IV c was tough on those with lesser algebra skills.

Honors Trigonometry Name Trig Ch 2

Copy the original problem. Per _____ Date 9/17/99

Convince *me* that **you** understand the concept.

No decimal fractions.

I

Determine the exact value of the following. Reduce fully.

Do not leave radicals in the denominator.: $\sec\left(\frac{-7\pi}{12}\right)$ **(10 pts)**

II

For each of the following functions, state the domain, range, period and state whether the function is odd, even, or neither. **(5 pts ea)**

A) $A(x) = \sin x$ B) $B(x) = \cos x$ C) $C(x) = \tan x$

D) $D(x) = \sec x$ E) $E(x) = \csc x$ F) $F(x) = \cot x$

III

State the sine, cosine, and tangent "add/subtract" formulas we use in class. **(10 pts tot)**

IV

Leaving the *LEFT SIDE UNTOUCHED*, **PROVE** the given identity. **(10 pts ea)**

A) $3\cos^2 x = \cos 2x + \cos^2 x + 1$

B) $\tan x = \dfrac{1 - \cos 2x}{\sin 2x}$

C) $1 + \dfrac{1}{\cos x} = \dfrac{\tan^2 x}{\sec x - 1}$

D) $\dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} = \tan 3x$

V

Given $\sin x = \frac{4}{5}$ and $\frac{\pi}{2} < x < \pi$. Find the exact value of $\csc 3x$. **(10 pts)**

EXTRA CREDIT ..... **5 pts** .....

Prove $\tan x$ has period π and that $\cos x$ does *not* have period π.

I CIRCLE SECRETS

Honors Trig Chapter 2
Mr. Mumaugh

A) For each of the following, draw a circle indicating the given. State the coordinates and radians:

1) 30° 2) 120° 3) 210° 4) 330° 5) 135° 6) 45° 7) 180°
8) 315° 9) 60° 10) 210° 11) 225° 12) 90° 13) 150° 14) 0°

B) For each of the following, draw a circle indicating the given. State the coordinates and degrees:

1) $\frac{\pi}{3}$ 2) $\frac{3\pi}{4}$ 3) $\frac{5\pi}{3}$ 4) 2π 5) $\frac{\pi}{6}$ 6) $\frac{\pi}{2}$ 7) $\frac{7\pi}{4}$

8) $\frac{\pi}{4}$ 9) $\frac{2\pi}{3}$ 10) $\frac{5\pi}{4}$ 11) $\frac{3\pi}{2}$ 12) $\frac{11\pi}{6}$ 13) $\frac{5\pi}{6}$ 14) 0

C) For each of the following, draw a circle indicating the given. State the radians and degrees:

1) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ 2) $(0, 1)$ 3) $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ 4) $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ 5) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

6) $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ 7) $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ 8) $(-1, 0)$ 9) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ 10) $(0, 1)$

11) $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ 12) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ 13) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ 14) $(1, 0)$ 15) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

II Write the general cosine add/subtract formula *EACH* time it is used!

A) Simplify: 1) $\text{Cos}\,\frac{5\pi}{12}$ 2) $\text{Cos}\,\frac{11\pi}{12}$ 3) $\text{Cos}\,\frac{13\pi}{12}$ 4) $\text{Cos}\,\frac{\pi}{12}$

5) $\text{Cos}\,\frac{-\pi}{12}$ 6) $\text{Cos}\,\frac{-13\pi}{12}$ 7) $\text{Cos}\,\frac{-11\pi}{12}$ 8) $\text{Cos}\,\frac{-5\pi}{12}$

B) WithOUT evaluating EACH factor in the original, evaluate: (hint: use Cos Add/Subtract)

1) $\cos\pi\cos\frac{2\pi}{3} + \sin\pi\sin\frac{2\pi}{3}$ 2) $\cos\frac{\pi}{6}\cos\frac{2\pi}{3} - \sin\frac{\pi}{6}\sin\frac{2\pi}{3}$

C) *USE the Cosine Add/Subtract formula* to **simplify as much as possible.**

1) $\cos(x + \pi)$ 2) $\cos(x - \pi)$ 3) $\cos\left(x + \frac{\pi}{2}\right)$ 4) $\cos\left(x - \frac{\pi}{2}\right)$ 5) $\cos(x + x)$

6) $\cos\left(\frac{x}{2} - \frac{x}{2}\right)$ 7) $\cos(0 - x)$ 8) $\cos\left(\frac{x}{2} + \frac{x}{2}\right)$ 9) $\cos\left(\frac{\pi}{2} - x\right)$

III Write the general sine add/subtract formula *EACH* time it is used!

Simplify: 1) $\sin\frac{5\pi}{12}$ 2) $\sin\frac{11\pi}{12}$ 3) $\sin\frac{13\pi}{12}$ 4) $\sin\frac{\pi}{12}$

5) $\sin\frac{-\pi}{12}$ 6) $\sin\frac{-13\pi}{12}$ 7) $\sin\frac{-11\pi}{12}$ 8) $\sin\frac{-5\pi}{12}$

IV

Draw the circle and properly mark the indicated radian measurement. Specifically state the sine of the measurement and the cosine of the measurement.

a) $\frac{5\pi}{6}$ b) $\frac{5\pi}{3}$ c) $\frac{-5\pi}{4}$ d) $\frac{-8\pi}{3}$ e) $\frac{13\pi}{12}$ f) $\frac{11\pi}{3}$ g) $\frac{-4\pi}{3}$ h) $\frac{-5\pi}{3}$

i) $\frac{11\pi}{2}$ j) $\frac{40\pi}{3}$ k) $\frac{-2\pi}{3}$ l) $\frac{-7\pi}{4}$ m) $\frac{11\pi}{6}$ n) $\frac{35\pi}{6}$ o) $\frac{-17\pi}{3}$ p) $\frac{-47\pi}{4}$

V

Copy each of the tables. Fill in all the missing data. Show all work. No decimals, of course.

A)

x	$\sin x$	$\cos x$	$\tan x$	x	$\sin x$	$\cos x$	$\tan x$
0				π			
$\frac{\pi}{6}$				$\frac{7\pi}{6}$			
$\frac{\pi}{4}$				$\frac{5\pi}{4}$			
$\frac{\pi}{3}$				$\frac{4\pi}{3}$			
$\frac{\pi}{2}$				$\frac{3\pi}{2}$			
$\frac{2\pi}{3}$				$\frac{5\pi}{3}$			
$\frac{3\pi}{4}$				$\frac{7\pi}{4}$			
$\frac{5\pi}{6}$				$\frac{11\pi}{6}$			

B)

x	$\csc x$	$\sec x$	$\cot x$	x	$\csc x$	$\sec x$	$\cot x$
0				π			
$\frac{\pi}{6}$				$\frac{7\pi}{6}$			
$\frac{\pi}{4}$				$\frac{5\pi}{4}$			
$\frac{\pi}{3}$				$\frac{4\pi}{3}$			
$\frac{\pi}{2}$				$\frac{3\pi}{2}$			
$\frac{2\pi}{3}$				$\frac{5\pi}{3}$			
$\frac{3\pi}{4}$				$\frac{7\pi}{4}$			
$\frac{5\pi}{6}$				$\frac{11\pi}{6}$			

VI

Copy the table. Fill in all the missing data. Show all work. No decimals, of course.

x	$\tan x$	x	$\tan x$	x	$\tan x$	x	$\tan x$
$\frac{\pi}{12}$		$\frac{9\pi}{12}$		$\frac{17\pi}{12}$		$\frac{25\pi}{12}$	
$\frac{2\pi}{12}$		$\frac{10\pi}{12}$		$\frac{18\pi}{12}$		$\frac{26\pi}{12}$	
$\frac{3\pi}{12}$		$\frac{11\pi}{12}$		$\frac{19\pi}{12}$		$\frac{27\pi}{12}$	
$\frac{4\pi}{12}$		$\frac{12\pi}{12}$		$\frac{20\pi}{12}$		$\frac{28\pi}{12}$	
$\frac{5\pi}{12}$		$\frac{13\pi}{12}$		$\frac{21\pi}{12}$		$\frac{29\pi}{12}$	
$\frac{6\pi}{12}$		$\frac{14\pi}{12}$		$\frac{22\pi}{12}$		$\frac{30\pi}{12}$	
$\frac{7\pi}{12}$		$\frac{15\pi}{12}$		$\frac{23\pi}{12}$		$\frac{31\pi}{12}$	
$\frac{8\pi}{12}$		$\frac{16\pi}{12}$		$\frac{24\pi}{12}$		$\frac{32\pi}{12}$	

VII

Prove:

a) $\sin^2\left(\frac{x}{2}\right) = \frac{1}{2}(1 - \cos x)$

b) $\left(\frac{1 + \tan x}{1 - \tan x}\right)^2 = \frac{1 + \sin 2x}{1 - \sin 2x}$

c) $\sin 3x = 3\sin x - 4\sin^3 x$

d) $\frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2\sin^2 x$

VIII

Which (if any) of the following are identities?

a) $\tan^2 x - \sin^2 x = (\tan^2 x)(\sin^2 x)$

b) $\frac{\cos 4x}{1 + \sin 4x} = \sec 4x - \tan 4x$

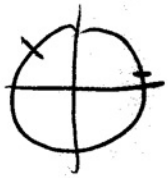
c) $\frac{1}{6}(\sin 4x)(\cos 2x) - \frac{1}{3}\cos 4x \sin 2x = \frac{1}{3}\sin^3 2x$

d) $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

I $\csc\left(-\frac{83\pi}{12}\right)$

a) $\frac{72+11}{12}$

$-\csc\frac{11\pi}{12}$



$\frac{-1}{\sin\frac{2\pi}{12}+\frac{2\pi}{12}}$

$\frac{-1}{\sin\frac{3\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{6}\cos\frac{3\pi}{4}}$

$\frac{-1}{\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{1}{2}\left(\frac{-\sqrt{2}}{2}\right)}$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

$\frac{4}{\sqrt{2}-\sqrt{6}}\cdot\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}+\sqrt{6}}$

$\frac{\cancel{4}(\sqrt{2}+\sqrt{6})}{-\cancel{4}}$

$-\sqrt{2}-\sqrt{6}$

b) $\tan 15^\circ =$

$\tan(45^\circ-30^\circ)$

$\tan\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$

$\frac{\tan\frac{\pi}{4}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{4}\tan\frac{\pi}{6}}$

$\frac{1-\frac{1}{\sqrt{3}}}{1+1\cdot\frac{1}{\sqrt{3}}}$

$\frac{\sqrt{3}-1}{1+\sqrt{3}}\cdot\frac{\sqrt{3}-1}{\sqrt{3}-1}$

$\frac{3-2\sqrt{3}+1}{2}$

$\frac{4-2\sqrt{3}}{2}$

$2-\sqrt{3}$

e) $\sec\left(\frac{2\pi}{3}+\left(\frac{5\pi}{6}+\frac{\pi}{2}\right)\right)$

$\frac{4\pi+5\pi+3\pi}{6}$

$\sec 2\pi$

1

II $\sin p=\frac{5}{13}$

$\cos p=\sqrt{1-\frac{25}{169}}=\frac{12}{13}$

$\sin q=\frac{-4}{5}$

$\cos q=\frac{-3}{5}$

a) $\sin(p+q)=\sin p\cos q+\sin q\cos p$

$=\frac{5}{13}\cdot\left(\frac{-3}{5}\right)+\left(\frac{-4}{5}\right)\left(\frac{12}{13}\right)$

$=\frac{-5}{65}-\frac{48}{65}$

$=\frac{-53}{65}$

b) $\cos(p+q)=$

$\cos p\cos q-\sin p\sin q$

$\frac{12}{13}\left(\frac{-3}{5}\right)+\left(\frac{5}{13}\right)\left(\frac{-4}{5}\right)$

$\frac{-36}{65}+\frac{20}{65}<0$ ∴ 3rd Quad.

Both sin and cos <0

III c)

$\sin(p-q)=$

$\sin p\cos q-\sin q\cos p$

$\frac{5}{13}\cdot\left(\frac{-3}{5}\right)-\left(\frac{-4}{5}\right)\left(\frac{12}{13}\right)$

$\frac{-15}{65}+\frac{48}{65}=\frac{33}{65}$

d) $\cos(p-q)=$

$\cos p\cos q+\sin p\sin q$

$\left(\frac{12}{13}\right)\left(\frac{-3}{5}\right)+\frac{5}{13}\left(\frac{-4}{5}\right)<0$ ∴ 2nd Quad.

III

1) $\csc x = \dfrac{\sin x}{1+\cos x} + \cot x$

$$\frac{\sin x}{1+\cos x} + \frac{\cos x}{\sin x}$$

$$\frac{\sin^2 x + \cos x + \cos^2 x}{\sin x\,(1+\cos x)}$$

$$\frac{1+\cos x}{\sin x\,(1+\cos x)}$$

$$= \frac{1}{\sin x}$$

$$= \csc x$$

B) $\dfrac{\sec x}{\sin x} = \tan x + \dfrac{1}{\tan x}$

$$= \frac{\tan^2 x + 1}{\tan x}$$

$$= \frac{\dfrac{\sin^2 x + \cos^2 x}{\cos^2 x}}{\dfrac{\sin x}{\cos x}}$$

$$= \frac{1}{\cos^2 x} \cdot \frac{\cos x}{\sin x}$$

$$= \frac{1}{\sin x \cos x}$$

$$= \frac{\sec x}{\sin x}$$

C) $2\csc x = \dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x}$

$$= \frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x\,(1+\cos x)}$$

$$= \frac{2\,\cancel{(1+\cos x)}}{\sin x\,\cancel{(1+\cos x)}}$$

$$= 2\csc x.$$

Honors Trigonometry Name _____

Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept!

Chapter 2 Make-Up Exam

I

Evaluate: (10 pts ea)

a) $\csc\left(-\frac{83\pi}{12}\right)$ b) $\tan 15^\circ$ c) $\sec\left(\frac{2\pi}{3} + \left(\frac{5\pi}{6} + \frac{\pi}{2}\right)\right)$

II

Given: $\sin p = \frac{5}{13}$ $0 < p < \frac{\pi}{2}$ and $\sin q = -\frac{4}{5}$ $\pi < q < \frac{3\pi}{2}$. (25 pts tot)

a) determine: $\sin(p + q)$

b) In which quadrant is $(p + q)$?

c) determine: $\sin(p - q)$

d) In which quadrant is $(p - q)$?

III

Leaving the left side untouched, prove: (15 pts ea)

a) $\csc x = \frac{\sin x}{1 + \cos x} + \cot x$

b) $\frac{\sec x}{\sin x} = \tan x + \frac{1}{\tan x}$

c) $2 \csc x = \frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x}$

We Don't Need No Stinkin' Pg 66

Open your text to page 66..... a wealth of information!. The first question that comes to mind is, "Why is this page *still* in the text book? .... Such an encyclopedia of information begs to be torn from the book and hung in a student's bedroom so he/she can see it upon awakening in the morning, just before drifting off to sleep at night and other inspirational times of the day.

But wait ....

Let's look closer .....

Section A ..... obviously!

Section B ..... $S^2 + C^2 = 1$ then divide by S^2 to get $1 + \cot^2 x = \csc^2 x$

or divide by $\cos^2 x$ to get $\tan^2 x + 1 = \sec^2 x$

We *already* know the Heston Identity so this section is obvious!

Section C ..... If I wanted to know what $\cos\left(\frac{\pi}{2} - x\right)$ was ....

I'd use the cosine add/subtract formula I already know! Obvious!

Section D ..... Obvious. Cosine and secant are even ... they "absorb the negative", the other trig functions are called odd functions.

Section E ..... This looks impressive but our version is more readable. We will use *our* version. Obviously!

Section F ..... See the comment for section C. Obvious.

Section G ..... Why memorize a zillion formulas? We know $2x = x + x$ and use the appropriate add identity. Obvious.

Section H ..... Useful ... on occasion. Think .... $\frac{x}{2} + \frac{x}{2} = x$ and you can generate these in a flash. Almost obvious.

So.... the conclusion is.....

We know $\sin^2 x + \cos^2 x = 1$

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos(A \pm B) = \cos A \cos B \pm \sin A \sin B$ and

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ and generally, that is all we really need to know.

What we really need is practice using these identities and we have plenty of that coming up right now...

The textbook offers two versions of "practice". **Verify** and **Prove** are the two types. The main difference between them is that when you **verify** an identity, you are allowed to manipulate both sides of the identity and in the case of a **Prove**, you are (generally) not allowed to alter one side or the other.

To make it less confusing, we *never verify*. We **always prove**. There are two distinct versions of **Prove**.

We Don't Need No Stinkin' Pg 66

Style 1 of **Prove** is to leave one side untouched ... make the other side absolutely identical to the untouched side using standard algebraic steps. This method is usually the easiest approach.
Style 2 of **Prove** is to begin with a known identity such as Heston's Identity and to manipulate both sides until the entire line ends up identical to the original problem.
One side-note is that it seems that the objective of the "problem writers" was to discover "similar but different" statements which don't look obvious. Some of us think they had too much time on their hands. In any case, they do supply us with problems for us to practice our knowledge of the basic trig formulas.

WTP stands for "Want To Prove". This tells you the objective of the problem.

Example of style 1:

WTP: $\dfrac{1-\sin x}{\cos x} = \dfrac{\cos x}{1+\sin x}$

$$= \frac{\cos x}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x}$$

$$= \frac{\cos x(1-\sin x)}{(1+\sin x)(1-\sin x)}$$

$$= \frac{\cos x(1-\sin x)}{1-\sin^2 x}$$

$$= \frac{\cos x(1-\sin x)}{\cos^2 x}$$

$$= \frac{1-\sin x}{\cos x}$$

Example of style 2:

WTP: $\sin x + \cos x \cot x = \csc x$

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{1}{\sin x} \cdot (\sin^2 x + \cos^2 x) = 1 \cdot \frac{1}{\sin x}$$

$$\sin x + \frac{\cos^2 x}{\sin x} = \frac{1}{\sin x}$$

$$\sin x + \cos x \cdot \frac{\cos x}{\sin x} = \frac{1}{\sin x}$$

$$\sin x + \cos x \cdot \cot x = \csc x$$

Your success with these problems depends more on your algebraic skills than it does on your trigonometric skills. Don't skip steps. Perform organized, orderly, legible algebraic operations. There is no doubt that your trig skills will improve. We hope your algebraic skills will be reinforced so your confidence level will be near "mastery" level. Do *honors level* work!

One final note...

The text refers to "restrictions" when they show proofs and verifies. We will ignore all references to "restrictions" and any references to "properties". We cover this material in greater depth in Chapter 4. For now, just concentrate on proving the given identities.